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Decision Support

Dynamic maintenance decision-making for series–parallel manufacturing system based on MAM–MTW methodology

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ABSTRACT

Proper maintenance schedule is required to improve manufacturing systems' profitability and productivity. A novel dynamic maintenance strategy is thus developed to incorporate both the single-machine optimization and the whole-system schedule for series–parallel system. Firstly, multiple attribute value theory and maintenance effects are considered in the single-machine optimization. A developed multi-attribute model (MAM) is used to determine the optimal maintenance intervals. Then, a series–parallel structure of the system is investigated in terms of the whole-system schedule. Maintenance time window (MTW) programming is presented to make a cost-effective system schedule by dynamically utilizing maintenance opportunities. The maintenance scheme achieved by using the proposed MAM–MTW methodology is demonstrated through a case study in a hydraulic steering factory. It is concluded that proper consideration of maintenance effects and time window leads to a significant cost reduction.

1. Introduction

In the current fast-paced industrial environment, more and more complex series–parallel manufacturing systems are used to satisfy production requirements. Unfortunately, unexpected failures have raised the cost of manufacturing process. In a survey of plant managers, Swanson (1999) concluded that more extensive use of maintenance was required for advanced manufacturing systems. The maintenance strategies require appropriate schedule methods to achieve high system performance with minimum cost. Therefore, the role and importance of maintenance schedule, as a decision-making process, has been increasingly recognized in many factories. However, about one third of the total maintenance costs were wasted due to unnecessary and improper maintenance activities (Mobley, 2002). There is a need to develop efficient maintenance strategy considering machine degradation and system structure to keep the system and its machines in good condition. It is clear that a dynamic schedule strategy in a series–parallel system is necessary to operate in a cost effective manner.

The machine condition deteriorates with usage and age over time. This degradation will inevitably lead to a failure and corresponding downtime, unless maintenance activity is performed. The reliability evolution of a system depends on its structure as well as the reliability evolution of its machines. Compared with the corrective maintenance, preventive maintenance (PM) shows that it is more positive and efficient (Wang, 2012). It is because that PM improves the machine condition before failures happen, the large loss caused by unexpected failures can be thus avoided. In the past several decades, many researchers have studied the analysis and modeling of maintenance operations to ensure safety and reliability, decrease frequency and severity of failures, reduce maintenance and breakdown cost, and improve availability (Zhou et al., 2007; Jin et al., 2009; Topal and Ramazan, 2010).

Today's series–parallel system usually consists of multiple machines, whether type or scale. With usage and age, all of them suffer increasing wear at various rates. A lot of maintenance models for single machine have been developed. However, two issues still need to be addressed. On one hand, traditional maintenance schedule usually suffers from a critical problem which sets periodic intervals to perform PM without considering machine condition. In fact, however, the assumption of perfect repair, which recovers a machine to an "as good as new" state, has been proved to be far from the truth. Even though some components are replaced, the cumulative wear on adjacent components may deteriorate unnoticed. Therefore, the practical maintenance effects have been discussed by Pham and Wang (1996). On the other hand, most developed maintenance models were focusing on cost. However, it would be better to consider other factors, such as availability, when developing a model to plan optimal PM intervals (Jiang and Ji, 2002; Zhu et al., 2010; Costa et al., 2012). For instance, Li et al. (2009b) proposed a two-stage approach for solving multi-objective system reliability optimization problems. A Pareto optimal solution set is initially identified at the first stage, and an

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Nomenclature

j	index of machine Sj, $j \in \{1,2,\ldots,J\}$
i	index of PM cycles in the single-machine schedule, $i \in \{1,2,\ldots,I\}$
T_d	system mission lifetime
$\lambda_{ij}(t)$	hazard rate function prior to the ith PM
C_{pij}	cost of PM action
T_{pij}	time duration of PM action
C_{fij}	cost of minimal repair
T_{fij}	time duration of minimal repair
A_{ij}	availability of the ith PM cycle for Sj
c_{rij}	cost rate of the ith PM cycle for Sj
T_{aij}	PM interval of availability model
T_{cij}	PM interval of cost model
T_{oij}	PM interval of the MAM
k	index of PM cycles in the whole-system schedule, $k \in \{1,2,\ldots,K\}$
t_{jk}	PM time point of Sj in whole-system scheduling
t_k	PM execution point in whole-system scheduling
$\Theta(j,t_k)$	maintenance decision for Sj at the time point t_k
T_w	width of maintenance time window
c_{dj}	downtime cost rate
ETC_{kj}	expected total cost of the kth cycle for Sj

integrated multiple objective selection optimization method is utilized at the second stage to determine a more systematic and meaningful recommended solution. Therefore, this study incorporates multiple attribute value theory and maintenance effects by presenting a multi-attribute model (MAM). This model not only takes availability and cost into account, but also considers imperfect maintenance effects for the single-machine schedule.

Recently, there has been a growing interest in modeling and optimization of multi-unit systems with the increasing demand from industry. Modern manufacturing systems are highly complex systems of closely interconnected machines (Dekker et al., 1997; Sun et al., 2008; Bedford et al., 2011). Therefore, any maintenance decision-making to be applied in such a system should consider not only one single machine degradation, but also system structure. Li (2009) has shown that the manufacturing systems are multi-unit systems where the system structure and buffers as well as the blockage and starvation times between subsystems and machines are interrelated. Modeling and understanding of these complex interactions not only represent a significant challenge, but also present maintenance decision makers with opportunities to perform maintenance while minimally affecting production (Li and Ni, 2009; Li et al., 2009a). In spite of the complexity of system maintenance scheduling, research related to this problem has been reported in the literature in term of different system structures. Marseguerra et al. (2002) studied optimal maintenance solutions for continuously monitored multi-component systems with Markov deteriorating processes. The Monte Carlo simulation was used for the optimization and it was more efficient than the analytical method. Tsai et al. (2004) gave an availability-centered preventive maintenance model for multi-unit systems, which was based on sequential PM theory. Zhou et al. (2009) proposed an opportunistic PM scheduling algorithm for the multi-unit series system. The optimal maintenance practices were determined by maximizing maintenance cost savings for the whole system. In addition, Ruiz-Castro and Li (2011) developed an algorithm for a discrete k-out-of-n system subject to several types of failure. The system was modelled and the stationary distribution was built by using matrix analytic methods. Tan et al. (2011) considered a parallel-machine scheduling problem with machine maintenance. The objective was to minimize the total completion time of all jobs.

In all, it shows that these works play a great role in maintenance scheduling for multi-unit systems. However, some of these strategies suffer from intractability when the number of machines grows. The traditional description of the system condition (i.e. Markov process) makes the analysis extremely complicated in series–parallel system modeling. Since the condition space in such problems grows exponentially with the number of machines, the Markov decision modeling is not tractable for more than three non-identical machines (Wildeman et al., 1997). Besides, maintenance schedule has a direct influence on production performance in a series–parallel system. In the notable studies from the Center for Intelligent Maintenance Systems in Cincinnati, Yang et al. (2007, 2008) pointed out that existing works have not systematically taken the on-line information into consideration in determining maintenance schedule. Wu et al. (2010) developed an online adaptive condition-based maintenance method for mechanical systems with a concentration on condition monitoring. Lee et al. (2011) discussed the state-of-the-art research in the areas of self-maintenance and engineering immune systems for machines with smarter adaptability to operating regime changes in future manufacturing systems. In sum, dynamic maintenance decision-making is imperative to enable manufacturing operations respond to the system degradation. Conventional maintenance decision support focuses on long-term statistic analysis, which is usually not applicable in a practical factory. Thus, there is a great need to propose a systematic methodology, which achieves a cost-effective system maintenance schedule according to the real-time single-machine schedule. Intelligent monitoring tools and e-maintenance techniques make it more feasible and economical viable to implement the preliminary data processing (Lee et al., 2006).

In this paper, a general maintenance decision-making strategy is proposed by considering both machine degradation and system structure. This policy helps assist a plant manager in making a dynamic maintenance plan based not only on the optimization of single-machine plan, but also on the global scheduling of whole-system programming. For a single machine, the intervals for PM actions are arranged through the MAM, which is established on multiple attribute value theory and maintenance effects. According to real-time single-machine schedule, the maintenance time window (MTW) programming is applied. A downtime caused by a machine could be used to perform PM on non-failed machines, while unnecessary breakdown of the whole system should be avoided. The aim is to systematically determine the system maintenance schedules that optimize the cost effect and decrease the decision-making complexity. The remained of this paper is organized as follows: In Section 2, the MAM–MTW methodology is described. In Section 3, the MAM for single machine is introduced and demonstrated. In Section 4, the MTW method for the series–parallel system based on dynamic programming is developed and then applied in a case study. The comparisons between the proposed maintenance strategy and various traditional strategies are discussed in Section 5. Finally, the conclusions and future work are enclosed in Section 6.

2. Research design and methodology

We apply the presented MAM–MTW methodology, as a decision-making process, in the dynamic maintenance strategy to achieve a cost-effective system maintenance schedule. This method dynamically utilizes the maintenance opportunities according to the single-machine schedule. Two kinds of maintenance action are considered to reduce unanticipated downtime. PM is imperfect maintenance, which does not make the machine be as good as new, but younger. Minimal repair is used on a machine if it fails between successive PM activities. This repair only recovers the machine to the failure rate it had when it failed. The scheme of the dynamic maintenance strategy is shown in Fig. 1.

The strategy consists of four layers in which the maintenance schedule is optimized along the way. The first layer is the physical layer, where a series–parallel system is defined as the decision object. The system structure, as well as the condition monitoring data and the historical event data of its machines, will be required and analyzed. In the second layer, the data about the system will be processed. The values of these data are used to determine the machine hazard rate evolution, the reliability parameters and the maintenance effects. Many data processing techniques have been studied in this layer (Niu and Yang, 2010; Carr and Wang, 2011). The third layer is the most important one for the plant manager. With the information gathered in the previous layers, one will have to decide upon the maintenance plan for the series–parallel system. We develop the MAM optimization for one single machine and the MTW programming for the whole system. The information transfer in this layer is not a "push" process, but a "pull" process. According to the schedule need for the whole system, pull the real-time PM intervals from the single-machine schedule. Thus, the MTW programming could dynamically utilize the maintenance opportunities based on the PM intervals of the machines in a short-term horizon. The fourth layer puts the decisions into practice. Maintenance actions execution and system performance evaluation are performed according to the system maintenance schedule layout. To validate the strategy, the MAM–MTW methodology will be introduced and demonstrated in detail.

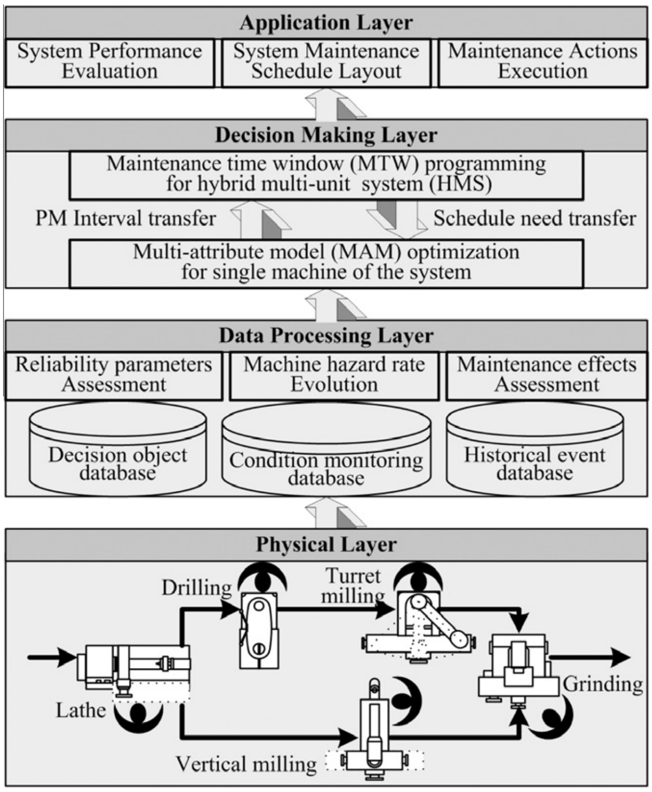

Fig. 1. Scheme of the dynamic maintenance strategy.

General assumptions:

(1) The system enters service at time $t = 0$. Its schedule is planed during the system mission lifetime. The entire system undergoes relatively constant conditions of stress, velocity and maintenance during this planning horizon.
(2) The hazard rate function is continuous and strictly increasing if there is no repair or PM. In practice, the rate of deterioration increases as the machine ages if there is no maintenance intervention. The hazard rate at time t reflects the health condition at this point of time.
(3) Setup times among the machines are negligible and infinite buffers exist between the machines. Thus, the production operations in the system continue except that the machine is not available due to PM, minimal repair or breakdown.
(4) The degradation of each machine is unrelated, since different kinds of machines suffer increasing wear at different rates as deterioration process. The machines of a series–parallel system are closely interconnected due to the system structure.

3. Multi-attribute model (MAM) for single machine

For the single-machine schedule, we define the PM cycle as the duration between two successive PM actions. Machine availability is considered as a local objective related to the efficiency, while maintenance cost rate is considered as another local objective related to the economy. Then, a multi-attribute model (MAM) based on multiple attribute value theory is proposed to give an overall objective to determine the optimal PM cycles. During the mission lifetime, this MAM optimization for Sj can be carried out and solved with the following steps:

Step 1: Assess the reliability parameters (including T_{d}, $T_{\mathrm{p}ij}$, $T_{\mathrm{f}ij}$, $C_{\mathrm{p}ij}$, $C_{\mathrm{f}ij}$), the maintenance effects and the initial hazard rate function $\lambda_{1j}(t)$ from the condition monitoring and the historical event databases in the data processing layer. Start the policy from the cycle $i = 1$, which means the initial PM cycle.

Step 2: Solve the availability model and the cost model, separately. The solutions are A^*_{ij}, $T^*_{\mathrm{a}ij}$, $c^*_{\mathrm{r}ij}$, $T^*_{\mathrm{c}ij}$. This is the process of single-objective scheduling of the ith PM cycle. In efficiency part, a PM cycle is divided into two time intervals: MUT is mean useful time, denoted by $T_{\mathrm{a}ij}$, under the availability model, while MDT is mean down time. The availability of the ith PM cycle for Sj can be represented as:

$$A_{ij} = \frac{\mathrm{MUT}}{\mathrm{MUT}+\mathrm{MDT}} = \frac{T_{\mathrm{a}ij}}{T_{\mathrm{a}ij} + \left(T_{\mathrm{p}ij} + T_{\mathrm{f}ij}\int_0^{T_{\mathrm{a}ij}} \lambda_{ij}(t)\mathrm{d}t\right)} \quad (1)$$

where the numerator equals to the MUT, and the denominator equals to the total duration. $\int_0^{T_{ij}} \lambda_{ij}(t)\mathrm{d}t$ is the expected frequency of the failures between successive PM activities. The optimal value of PM interval $T^*_{\mathrm{a}ij}$ corresponding to the maximum A^*_{ij} is given by:
$\left.\frac{dA_{ij}}{dT_{\mathrm{a}ij}}\right|_T = 0$, that is

$$T_{\mathrm{p}ij} + T_{\mathrm{f}ij}\int_0^{T_{\mathrm{a}ij}} \lambda_{ij}(t)\mathrm{d}t - T_{\mathrm{f}ij}\cdot T_{\mathrm{a}ij}\cdot \lambda_{ij}(T_{\mathrm{a}ij}) = 0 \quad (2)$$

In economical part, maintenance cost contains of the cost of PM action and the possible cost of minimal repair. Let $T_{\mathrm{c}ij}$ be the PM interval under the cost model. The maintenance cost rate of the ith PM cycle can be represented as:

$$c_{\mathrm{r}ij} = \frac{C_{\mathrm{p}ij} + C_{\mathrm{f}ij}\int_0^{T_{\mathrm{c}ij}} \lambda_{ij}(t)\mathrm{d}t}{T_{\mathrm{c}ij} + T_{\mathrm{p}ij} + T_{\mathrm{f}ij}\int_0^{T_{\mathrm{c}ij}} \lambda_{ij}(t)\mathrm{d}t} \quad (3)$$

where the numerator equals to the total maintenance cost, and the denominator equals to the total duration. The optimal value of PM interval T^*_{cij} corresponding to the minimum c^*_{rij} is given by:

$$\left.\frac{dc_{rij}}{dT_{cij}}\right|_T = 0, \quad \text{that is}$$

$$\lambda_{ij}(T_{cij})(C_{fij}\cdot T_{cij} + C_{fij}\cdot T_{pij} - C_{pij}\cdot T_{fij}) - C_{fij}\int_0^{T_{cij}}\lambda_{ij}(t)\mathrm{d}t - C_{pij} = 0 \tag{4}$$

Step 3: Substitute the solutions (A^*_{ij} and c^*_{rij}) from Step 2 into the MAM, and solve it by minimizing the overall objective function. The solution is the optimal PM interval T^*_{oij}. This is the process of multi-objective optimal scheduling for the ith PM cycle.

Combining above two single-objective models together, we propose a MAM based on multiple attribute value theory to give an overall objective. Since a large value of A_{ij} is preferred and a small value of c_{rij} is preferred, it is necessary to unify the problem. We choose to minimize the overall objective, denoted by V_{ij}. Therefore, the expression $-A_{ij}/A^*_{ij}$ is utilized in the MAM. The overall objective function is thus defined as:

$$V_{ij} = -w_{1ij}\frac{A_{ij}}{A^*_{ij}} + w_{2ij}\frac{c_{rij}}{c^*_{rij}} \tag{5}$$

where w_{1ij} and w_{2ij} ($w_{1ij} \geqslant 0$, $w_{2ij} \geqslant 0$, $w_{1ij} + w_{2ij} = 1$) are weights of machine availability and maintenance cost, respectively. The relative importance of the two objectives is measured by the weight ratio. In practice, there are lots of methods proposed to determine these objective weights, such as Delphi method, Analytic Hierarchy Process (AHP), Entropy method and Fuzzy Cluster Analysis (Cheng et al., 1999). The case of $w_{1ij} = w_{2ij} = 0.5$ is just taken as an example.

In this function, the PM interval under the MAM, denoted by T_{oij}, takes place of T_{aij} and T_{cij}. The optimal PM interval T^*_{oij} can be obtained by minimizing the overall objective. We can have $\min(T^*_{aij}, T^*_{cij}) \leqslant T^*_{oij} \leqslant \max(T^*_{aij}, T^*_{cij})$ by: $\left.\frac{dV_{ij}}{dT_{oij}}\right|_T = 0$, that is

$$w_{2ij}\left[\lambda_{ij}(T_{oij})(C_{fij}\cdot T_{oij} + C_{fij}\cdot T_{pij} - C_{pij}\cdot T_{fij}) - C_{fij}\int_0^{T_{oij}}\lambda_{ij}(t)\mathrm{d}t - C_{pij}\right] - w_{1ij}\left[T_{pij} + T_{fij}\int_0^{T_{oij}}\lambda_{ij}(t)\mathrm{d}t - T_{fij}\cdot T_{oij}\cdot\lambda_{ij}(T_{oij})\right] = 0 \tag{6}$$

Step 4: Identify whether cumulative PM interval is beyond the system mission lifetime $(0, T_d]$. If no, go to Step 5 and plan the next PM cycle. Otherwise, go to Step 6 and end the scheduling for Sj.

Step 5: Introduce the hybrid hazard rate evolution based on the maintenance effects to describe the hazard rate of the next PM. Then assign $i = i + 1$ and turn back to Step 2 to plan the next cycle.

Given the fact that PM not only decreases the hazard rate to a certain value but also changes the slope of the hazard rate function, the relationship between those hazard rates before and after the ith PM is defined as:

$$\lambda_{(i+1)j}(t) = b_{ij}\lambda_{ij}(t + a_{ij}T_{ij}) \tag{7}$$

where $t \in (0, T_{(i+1)j})$. The age reduction factor $0 < a_{ij} < 1$ shows that imperfect PM makes machine's initial failure rate become $\lambda_{ij}(a_{ij}T_{ij})$ for the next cycle. Meanwhile, the

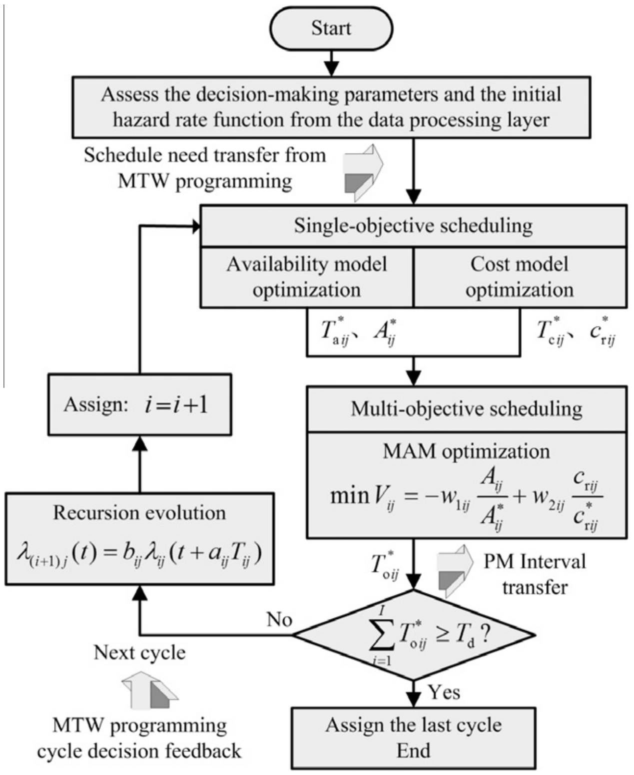

Fig. 2. MAM method for the single-machine scheduling.

hazard rate increase factor $b_{ij} > 1$ indicates that PM increases the failure rate $b_{ij}\lambda_{ij}(t)$ due to deterioration process. These factors can be determined from the condition monitoring and the historical event databases (Zhou et al., 2007).

Step 6: Assign the last PM cycle $T_{oIj} = T_d - \sum_{i=1}^{I-1}(T^*_{oij} + T_{pij} + T_{fij}\int_0^{T^*_{oij}}\lambda_{ij}(t)\mathrm{d}t)$. The flowchart of the single-machine schedule is shown in Fig. 2.

Based on the optimal PM intervals, the measure of the machine performance during the whole mission lifetime in efficiency part and in economical part can be separately acquired from the total machine availability (TA) and the total maintenance cost rate (Tc_r):

$$TA_j = \left(\sum_{i=1}^{I-1} T^*_{oij} + T_{oIj} - T_{fij}\int_0^{T_{oIj}}\lambda_{Ij}(t)\mathrm{d}t\right)\Big/ T_d \tag{8}$$

$$Tc_{rj} = \left[\sum_{i=1}^{I-1}(C_{pij} + C_{fij}\int_0^{T^*_{oij}}\lambda_{ij}(t)\mathrm{d}t) + C_{fij}\int_0^{T_{oIj}}\lambda_{Ij}(t)\mathrm{d}t\right]\Big/ T_d \tag{9}$$

The index of TA reflects the utilization level of Sj, while Tc_r reflects the overhead cost of the machine with the scheduled optimal PM intervals. The PM intervals will be used to support the MTW programming for the whole-system schedule.

3.1. Example of single machine

A series–parallel system in a hydraulic steering factory (Fig. 1) is selected as an example for numerical experiments using the proposed MAM–MTW methodology. The system consists of five different machines: S1 (Lathe machine), S2 (Drilling machine), S3 (Turret milling machine), S4 (Vertical milling machine) and S5 (Grinding machine). In this $J = 5$ manufacturing system, a special case assumed in which the hazard rate function for each machine is a Weibull function $\lambda_{1j}(t) = (m_j/\eta_j)(t/\eta_j)^{m_j-1}$, which is widely used to fit repairable equipment. For the purpose of performing a numerical investigation about the dynamic maintenance strategy, we present the parameters for each machine in Table 1.

To evaluate the performance of the single-machine schedule, S1 is taken as an example. Using the MAM method, the maintenance schedule for S1 is derived and the performance is evaluated. By running simulation over T_d = 25,000 hours, the schedule results for S1 are listed in Table 2. When $w_{1ij} = 1$, $w_{2ij} = 0$, the MAM becomes the availability model, and when $w_{1ij} = 0$, $w_{2ij} = 1$, it becomes the cost model. The optimal T^*_{oi1} in bold are scheduled in the case of $w_{1ij} = w_{2ij} = 0.5$. These PM intervals will be transferred to the whole-system schedule.

4. Maintenance time window (MTW) for whole system

A series–parallel system consists of different kinds of machines (units), which suffer increasing degradation at different rates with usage and age. The whole-system schedule not only depends on the real-time maintenance schedule of each machine, but also intensively depends on the system structure, which is determined by the production requirements. Chang et al. (2007) suggested that maintenance of a multi-component system differs from that of a single-unit system because of dependencies in multi-component systems. An opportunity arises if the failure of some other part of the system allows the component in question to be replaced. Alardhi and Labib (2008) studied the maintenance scheduling process as an optimization problem and the maintenance and system constraints include the crew constraint, maintenance window constraint and time limitation constraint. To achieve an effective system maintenance schedule, we propose a systematic methodology to incorporate available information about machine degradation and system structure. The maintenance time window (MTW) is defined as a criteria to separate the PM actions in parallel subsystems, while it is used to combine the PM actions in series subsystems together. The MTW optimum in series–parallel system aims to reduce the total system maintenance cost. Since this method dynamically utilizes the maintenance opportunities and avoids unnecessary downtimes resulting from excessive maintenance actions.

System mission lifetime T_d is divided into K cycles, each with PM execution point t_k at the cycle end. The sequence of PM executions should consider connections of all subsystems, and their individual machines' PM intervals as well. In a series–parallel system, some machines are connected in series, while several others are in parallel. Hybrid structure of the multi-unit manufacturing system is investigated. The MTW programming is thus developed to assist a plant manager in making a whole-system maintenance schedule to evaluate the system total cost. The schedule is based on the single-machine information in a short-term horizon.

4.1. MTW-separation in the parallel subsystem

For an N-unit parallel subsystem, PM actions performed on all its units at the same time means a breakdown of the whole system and unnecessary downtime of others. This situation should be avoided in the whole-system schedule. Thus, the MTW is defined as the criteria to separate the PM actions in the subsystem. The following procedure is the MTW programming in parallel subsystem.

1. Pull the real-time PM intervals from the single-machine schedule and evaluate the MTW value T_w ($T_w > \forall T_{pij}$). Start from the cycle $i = 1$, $k = 1$.
2. Assign the PM intervals $T^*_{oij} (0 < j \leqslant N)$ to the PM time points t_{jk} of each unit for the whole-system scheduling. In the first cycle, the time points are given by:

$$t_{jk} = T^*_{oij}, \quad i = 1, \ k = 1 \tag{10}$$

3. Check time point arrangement: Choose the first unit that reaches its PM interval as $j = m1$ in the kth cycle for the subsystem. Assign $T_{pk(m1)} = T_{pi(m1)}$. The check moment can be chosen with the following expression:

$$t_k = t_{(m1)k} = \min(t_{jk}), \quad 0 < j \leqslant N \tag{11}$$

4. Mission lifetime check: Identify whether the check time point is greater than or equal to T_d. If yes, the system mission life is over, go to Step (9) and end the scheduling. Otherwise, go to Step (5) and progress MTW-separation check.

Table 1
Machine parameters.

j	m_j	η_j	T_{pij}	T_{fij}	C_{pij}	C_{fij}	c_{dij}	a_{ij}	b_{ij}
1	3.0	8000	140	600	5000	35,000	80	$i/(15i+5)$	$(17i+1)/(16i+1)$
2	2.0	7000	120	200	6000	18,000	40	0.03	1.04
3	1.5	12,000	200	350	2000	15,000	30	$i/(20i+20)$	1.03
4	3.0	13,000	80	300	7500	22,000	45	0.025	$(16i+3)/(15i+3)$
5	2.5	16,000	300	800	2500	25,000	75	$i/(16i+14)$	1.05

Table 2
Optimal PM intervals for S1 under different weights.

PM cycle i	Availability model (1,0)			Cost model (0,1)			Multi-attribute model (MAM) (0.5,0.5)		
	T^*_{ai1}	A^*_{i1}	c^*_{ri1}	T^*_{ci1}	A^*_{i1}	c^*_{ri1}	T^*_{oi1}	A^*_{i1}	c^*_{ri1}
1	3909	0.9490	2.2052	3292	0.9477	2.1414	**3319**	0.9478	2.1415
2	3712	0.9465	2.3157	3125	0.945	2.2485	**3152**	0.9452	2.2487
3	3497	0.9434	2.4503	2942	0.9418	2.3790	**2969**	0.942	2.3792
4	3298	0.9401	2.5888	2774	0.9385	2.5133	**2801**	0.9387	2.5135
5	3117	0.9369	2.7300	2620	0.9352	2.6501	**2647**	0.9354	2.6503
6	2951	0.9336	2.8735	2479	0.9318	2.7891	**2506**	0.9320	2.7894
7	2798	0.9302	3.0193	2349	0.9283	2.9304	**2376**	0.9285	2.9308
8	248	0.9998	0.0134	2229	0.9248	3.0739	**2257**	0.925	3.0743
9	N/A	N/A	N/A	1711	0.9870	0.7604	**1498**	0.9927	0.5759

5. MTW-separation check: Identify whether for all other units $t_{jk} \leqslant t_k + T_{pk(m1)}(0 < j \leqslant N, j \neq m1)$, which means all units will be maintained at the same time. If yes, choose another $S_{m2}(m2 \neq m1)$, go to Step (7) for PM separation. Otherwise, go to Step (6), meanwhile take S_{m1} to Step (8) for PM execution.
6. For the next cycle, assign $k = k + 1$, $t_{jk} = t_{j(k-1)}$, $(j \neq m1, j \neq m2)$. Turn to Step (3).
7. PM separation: Separate the PM action of S_{m2} according to the MTW, thus $t_{(m2)k} = t_k + T_w$. Feedback this MTW-separation decision to the single-machine schedule of S_{m2}. Then, for the next cycle, assign $k = k + 1$, $t_{jk} = t_{(m2)(k-1)}$. Turn to Step (3).
8. PM execution: Execute the PM action of S_{m1}. For the next cycle, assign $k = k+1,\ i = i+1,\ t_{jk} = t_{(m1)(k-1)} + T_{p(k-1)(m1)} + T^*_{oi(m1)}$. Turn to Step (3).
9. End the MTW-separation programming.

4.2. MTW-combination in the series subsystem

For an M-unit series subsystem, when a PM action is performed on one machine, maintenance opportunities arise for other ones. It is often reasonable to assume that maintaining more than one machine at the same time can be more cost-effective than maintaining them separately. To combine the PM actions according to the system structure, the MTW also provides a criterion to schedule the whole-system maintenance plan. The following procedure determines the system maintenance schedule based on the MTW method.

1. Pull the real-time PM intervals from the single-machine schedule and evaluate the MTW value $T_w(T_w < \forall T^*_{oij})$. Start from the cycle $i = 1$, $k = 1$.
2. Assign the PM intervals $T^*_{oij}(0 < j \leqslant M)$ to the PM time points t_{jk} of each unit for the whole-system scheduling. In the first cycle, the time points are given by:

$$t_{jk} = T^*_{oij}, \quad i = 1,\ k = 1$$

3. PM of a unit creates opportunities for other units. The PM combination moment for the whole system can be chosen with the following expression:

$$t_k = \min(t_{jk}), \quad 0 < j \leqslant M \tag{12}$$

4. Mission Lifetime check: Identify whether the PM combination moment is greater than or equal to T_d. If yes, the system mission life is over, go to Step (7) to end the scheduling. Otherwise, go to Step (5) and progress MTW-combination check.
5. MTW-combination check: Identify whether the other units $j \in \{1, 2, \ldots, M\}$ are predicted to reach their PM time points within $[t_k, t_k + T_w]$. Accordingly, schedule the maintenance decision for unit j at the time point t_k, the following definition is used:

$$\Theta(j, t_k) = \begin{cases} 0 & t_{jk} > t_k + T_w \\ 1 & t_{jk} \leqslant t_k + T_w \end{cases} \tag{13}$$

where $\Theta(j,t_k) = 0$ means no maintenance action is initiated on unit j. On the contrary, $\Theta(j,t_k) = 1$ means the PM action is combined to perform in advance, then assign $t_{jk} = t_k$ to implement.
6. For the next cycle, assign $k = k + 1$, the new PM time points t_{jk} $(0 < j \leqslant M)$ are given by the following dynamic programming equation:

$$t_{jk} = \begin{cases} t_{j(k+1)} + T_{\mathrm{p}(k-1)\max} & \Theta(j, t_{k-1}) = 0 \\ t_{(k-1)} + T_{\mathrm{p}(k-1)\max} + T^*_{oij}(i = i + 1) & \Theta(j, t_{k-1}) = 1 \end{cases} \tag{14}$$

where $T_{p(k-1)\max}$ is the maximum time for PM actions combined in the last cycle, which is also the down time for the whole system during $[t_{k-1}, t_{k-1} + T_{p(k-1)\max}]$. Feedback this MTW-combination decision to the single-machine schedule of the combined units. Then turn back to Step (3) for scheduling the next PM combination moment.
7. End the MTW-combination programming. Output all the PM combination moments $t_k(0 < t_k \leqslant T_d)$ as the maintenance schedule.

4.3. MTW optimum in the series–parallel system

Dynamically applying the procedures presented in the previous section and performing PM activities according to the MTW programming based on real-time single-machine schedules, plant manager can obtain MTW optimum in series–parallel system. The interactive MAM–MTW methodology obtains the system maintenance schedule layout:

(1) *MAM optimization for each machine*: The intervals of the PM actions are arranged dynamically through the MAM method. These real-time decisions will be transferred to supply the dynamic whole-system schedules, which will feedback the cycle decisions from the MTW programming.
(2) *MTW-separation in parallel subsystem*: Based on real-time single-machine schedule, the MTW is defined as the criteria to separate the PM actions of the parallel machines to avoid the breakdown of the whole system. The decision will be transferred for the single-machine schedule in next cycle and for the related MTW-combination.
(3) *MTW-combination in series subsystem*: Given the information from MAM optimization and MTW-separation cycle by cycle, the MTW provides a criterion to combine the PM actions when a PM action is performed on one machine. This moment means maintenance opportunities arise for other machines in series. The decision will also be made and the PM combination moments $t_k(0 < t_k \leqslant T_d)$ will be obtained as the system maintenance schedule layout.
(4) *System performance evaluation*: The aim of MAM–MTW methodology is to achieve a cost-effective system maintenance schedule. The system performance by using the proposed methodology is evaluated based on the maintenance schedule layout. Let c_{dj} be the downtime cost rate, $T_{pk\max}$ be the maximum time for PM actions combined in the cycle, $T^*_{oij} - (t_{jk} - t_k)$ means the effective interval for advanced PM cycle. The total maintenance cost of the kth cycle for unit j can be evaluated by:

$$ETC_{kj} = \begin{cases} c_{dj} \cdot T_{pk\max} & \Theta(j, t_k) = 0 \\ C_{\mathrm{p}ij} + C_{\mathrm{f}ij} \int_0^{T^*_{oij} - (t_{jk} - t_k)} \lambda_{ij}(t)\mathrm{d}t + c_{dj} \cdot T_{pk\max} & \Theta(j, t_k) = 1 \\ 0 & \Theta(j, t_k) = 2 \end{cases} \tag{15}$$

where $\Theta(j,t_k) = 0$ means no maintenance action is initiated on unit j but the unit will be down; $\Theta(j,t_k) = 1$ means the PM action is combined to perform in advance; $\Theta(j,t_k) = 2$ means no maintenance action is initiated on unit j and the unit continues to operate. Thus, the total system maintenance cost for the system in its mission lifetime can be defined as:

$$ETC = \sum_{k=1}^{K} \left(\sum_{j=1}^{J} ETC_{kj} \right) \tag{16}$$

It should be noticed that the choice of MTW value directly impacts on MTW-separation in parallel subsystem and MTW-combination in series subsystem. This influences how much maintenance cost is required. Therefore, it is important to find the suitable T_w to reach the cost-effective whole-system schedule. The previous

Table 3
System maintenance schedule with T_w = 800 hours.

j	Time point of PM activity (hours)											
1	3319		6911	10,020	13,121		15,134	17,940		20,212		22,789
2	3319		6911	10,020			15,134		19,105			22,789
3		5108		10,020			15,134			20,212		
4			6911			14,455					21,984	
5		5108		10,020			15,134			20,212		

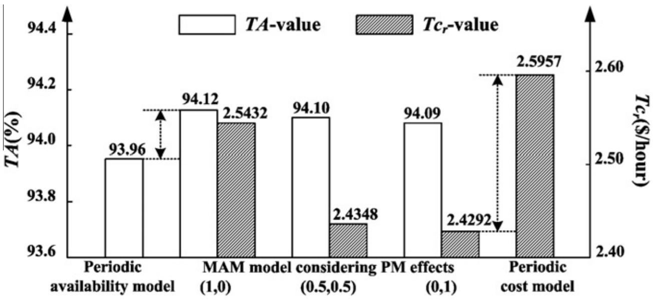

Fig. 3. Results comparison in the single-machine schedule.

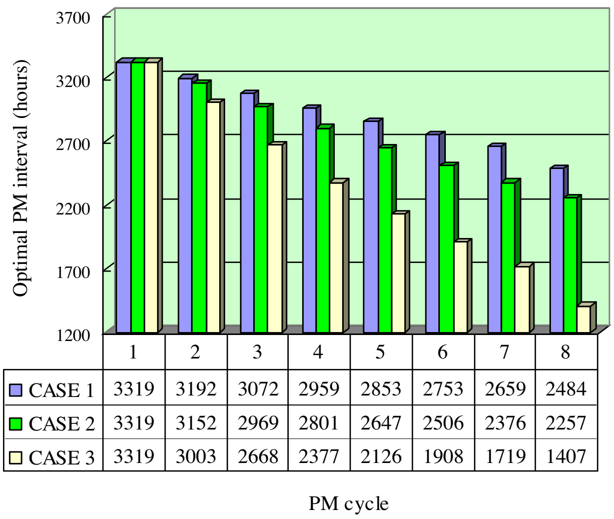

Fig. 4. PM schedules for various maintenance effects.

restriction $\forall T^*_{oij} > T_w > \forall T_{pij}$ ensures no interactive conflict between MTW-separation and MTW-combination. By minimizing *ETC* in Eq. (16), the suitable MTW for the system maintenance schedule layout can be determined.

Moreover, for traditional opportunistic maintenance policy which calculates the cost-savings of all possible combinations in the system at every time point of PM activity, its complexity for maintenance scheduling is $O(2^{(J-1)})$, which means the complexity grows exponentially with the number of machines. For the presented MTW method, its complexity is just polynomial with J, thus another advantage of our decision-making strategy is that a system consists of many machines can be handled.

4.4. Example of whole system

A simple series–parallel system consisting of five machines, as shown in Fig. 1, is used for simulation of the MTW programming described above. For each machine, a distribution describing the hazard rate on that machine over time. Parameters are listed in Table 1. The optimal PM intervals of these machines can be obtained in the single-machine schedule like S1. To achieve a cost-effective system maintenance schedule, the system structure is analyzed for the MTW programming. The composition operators $\oplus$ and $\otimes$ are defined for the parallel and series connections of the machines. It can be noticed there are four subsystems in that five-machine system: two parallel subsystems (S2 $\oplus$ S4; S3 $\oplus$ S4) and two series subsystems (S1 $\otimes$ S2 $\otimes$ S3 $\otimes$ S5; S1 $\otimes$ S4 $\otimes$ S5).

To validate the systematic methodology incorporating available information about machine degradation and system structure, we program the system maintenance schedule through the MTW method. Taken T_w = 800 for the MTW programming as an example, the system mission lifetime is 25,000 hour. Table 3 gives the system maintenance schedule layout under the relatively constant working condition. According to MTW schedule, in the fist cycle k = 1, S1 and S2 are maintained, where S2 is maintained in advance. At time 10,020 and 15,134, the PM actions are performed on the series subsystem (S1 $\otimes$ S2 $\otimes$ S3 $\otimes$ S5) at the same time. The influence of MTW-value and the effectiveness of MTW programming will be further discussed in Section 5.

5. Discussion

To validate the proposed MAM–MTW methodology, and evaluate this decision-making process in the dynamic maintenance

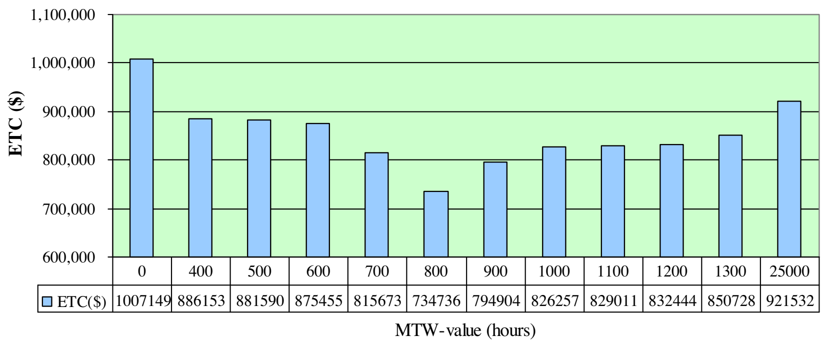

Fig. 5. *ETC* of the system with various MTW.

Table 4
System maintenance schedule with T_w = 600 hours.

j	Time point of PM activity (hours)														
1	3319			6911			10,020		12,980	15,274		17,980	20,132	21,954	
2		4181				8646			12,980		16,607		20,132		24,049
3			5228				10,020			15,274			20,132		
4					7788					15,274				21,954	
5			5228					11,014			16,607			21,954	

Table 5
System maintenance schedule with T_w = 1000 hours.

j	Time point of PM activity (hours)											
1	3319		6911	10,020	13,121		15,134	16,992	19,508		22,065	24,351
2	3319		6911	10,020	13,121			16,992		20849		24,351
3		5108		10,020			15,134		19,508			24,351
4			6911			14,455					22,065	
5		5108		10,020			15,134		19,508			24,351

strategy, we perform some further investigations on both the single-machine optimization and the whole-system schedule for the series–parallel system.

5.1. Experiment 1: Effectiveness of MAM optimization

The results of the MAM method in Table 2 reveal the following:

(1) The PM interval decreases (while A^*_{i1} decreases and c^*_{ri1} increases) as cycle i increases. This indicates that in the realm of aging, the underlying hazard rate evolution becomes faster with the machine ages, which proves that the machine is subject to a degradation process.
(2) It is apparent that the machine availability will be lower and the maintenance cost will be higher as the deteriorating machine ages. Such a trend occurs due to our introduction of maintenance effects. When the hazard rate increases, the increase of the downtime decreases the availability, while the addition of the failure frequency increases the cost.
(3) In the same PM cycle, A^*_{i1} under the availability model (1,0) is the highest while c^*_{ri1} under the cost model (0,1) is the lowest. Since the availability model focuses on maximizing availability and the cost model focuses on minimizing cost, while the MAM (0.5,0.5) takes both factors into consideration.

The measurements of S1 performance during the mission lifetime in efficiency part and in economy part are separately acquired from the indices of TA and Tc_r. The comparison of the results under the MAM method and two Periodic PM models is shown in Fig. 3. In Periodic availability model, the PM is performed with the same interval T_{aij} = 3909 from Eq. (2) without considering maintenance effects. Similarly, in Periodic cost model, the fixed PM interval is T_{cij} = 3292 from Eq. (4). It is clear that the values of TA under MAM models are higher than Periodic availability model, while the values of Tc_r are lower than Periodic cost model. The results indicate that ignoring the effects of a maintenance activity will lead to less availability and extra cost. The proposed MAM model contributes to more optimality of the PM intervals.

5.2. Experiment 2: Sensitivity study on maintenance effects

To investigate how maintenance effects of PM action affect the optimal PM intervals, we consider three cases of various situations. Case 1: a_{i1} = 0.04 and b_{i1} = 1.04, the factors are constant and small; Case 2: $a_{i1} = i/(15i+5)$ and $b_{i1} = (17i+1)/(16i+1)$, which means worse maintenance effects; Case 3: $a_{i1} = i/(9i+5)$ and $b_{i1} = (19i+1)/(16i+1)$, which describes the most steep hazard rate function. Fig. 4 shows the optimal PM intervals T^*_{oi1} of all three cases.

It is clear that the optimal PM intervals tend to decrease more rapidly as a_{i1} and b_{i1} increases. This signifies that an increase of a_i represents a decrease in the effectiveness of each PM, because the residual hazard rate after a PM, $\lambda_i(a_iT_i)$, increases. It is same to an increase in the value of b_i, since the increase rate of the hazard rate function becomes much higher. The correct description of PM effect can contribute to the optimality of the PM intervals.

5.3. Experiment 3: Whole-system schedule with different MTW

As mentioned before, the value of the MTW directly impacts on the whole-system schedule. To see how the system maintenance programming is influenced, various values of the MTW are investigated. Here, same set of parameters as that with T_w = 800 is used, except that T_w = 600 and T_w = 1000 are applied for the schedule. Tables 4 and 5 show the results obtained with various MTW values.

From the schedule result with T_w = 600 given in Table 4, it is visible that a shorter time window will lead to more machines maintained individually. For example, in the first cycle, the PM actions of S1 and S2 are not combined; this causes unnecessary downtimes for S3 at time 4181. If the PM is conducted on a machine only when it reaches its optimal intervals, unnecessary downtimes will inevitably increase the total system maintenance cost. It is favorable to include as many PM actions as possible according to the MTW methodology to save the ETC.

From the schedule result with T_w = 1000 given in Table 5, one can see that a longer time window can make more machines maintained at the same time. More PM actions performed on the machines at maintenance opportunities can decrease unnecessary downtimes. However, too many machines maintained in advance will lead to more PM actions, thus ETC will increase due to extra maintenance. To sum up, neither too long nor too short MTW should be applied. The suitable value of T_w is essential to reach the cost-effective whole-system schedule.

5.4. Experiment 4: Effectiveness of MTW programming

The aim of the MAM–MTW methodology is to achieve a cost-effective system maintenance schedule. The system performance

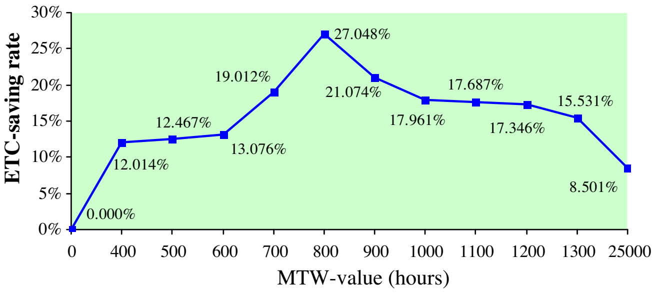

Fig. 6. *ETC*-saving rate with various MTW.

ETC is evaluated based on the maintenance schedule layout. The *ETC* values with various MTW and corresponding *ETC*-saving rates are shown in Figs. 5 and 6 respectively. Two other common maintenance scheduling methods are compared to validate the MTW method. One is that PM is conducted on a machine individually only when it reaches its intervals, which means $T_w = 0$. The other is that when one of the machines reaches its intervals, PM actions are carried out on all machines simultaneously, which in fact is $T_w = 25{,}000$.

Based on the results in Figs. 5 and 6, it can be found that *ETC* of $T_w = 0$ is 1,007,145, which is defined as the baseline. When the time window increases from 400 to 800, *ETC* decreases to 734,736 and *ETC*-saving rate increases to 27%. It is because that the longer MTW enables more machines to utilize the maintenance opportunities. However, when the time window extends to 1300, *ETC* value increases and *ETC*-saving rate increases. This means too long MTW incurs extra maintenance and more PM cost is needed. If $T_w = 25{,}000$, $ETC = 921{,}532$ is much higher than the presented MTW method. Obviously, $T_w = 800$ is the most suitable time window to obtain a cost-effective maintenance schedule for this five-machine system.

6. Conclusions

A general maintenance decision-making strategy for series–parallel system based on the MAM–MTW methodology is presented in this paper. This cost-effective method systematically considers both machine degradation and system structure according to the machine information in a short-term horizon. The aim is to obtain a dynamic maintenance plan based not only on the optimization of the single-machine plan, but also on the global scheduling of the whole-system programming. The developed MAM optimization considering multiple attribute value theory and maintenance effects is used to schedule the optimal maintenance intervals for single machines of the system. Furthermore, the MTW programming is presented to achieve a cost-effective system maintenance schedule by utilizing the maintenance opportunities dynamically according to the real-time single-machine plan. The cost savings achieved by using the MAM–MTW methodology is demonstrated through a case study. Results indicate that without proper consideration of maintenance effects and time window is likely to cause unanticipated downtimes or maintenance wastes. It can be concluded that the proposed MAM–MTW methodology leads to a significant cost reduction.

Nevertheless, industrial implementation and demonstration of the newly proposed method in various environments remains to be done in the future. The system maintenance schedule layout can be used to prepare the maintenance activities in advance to ensure the manufacturing production. Furthermore, how to introduce the manufacturing buffers into the MAM–MTW methodology will be investigated in future studies.

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